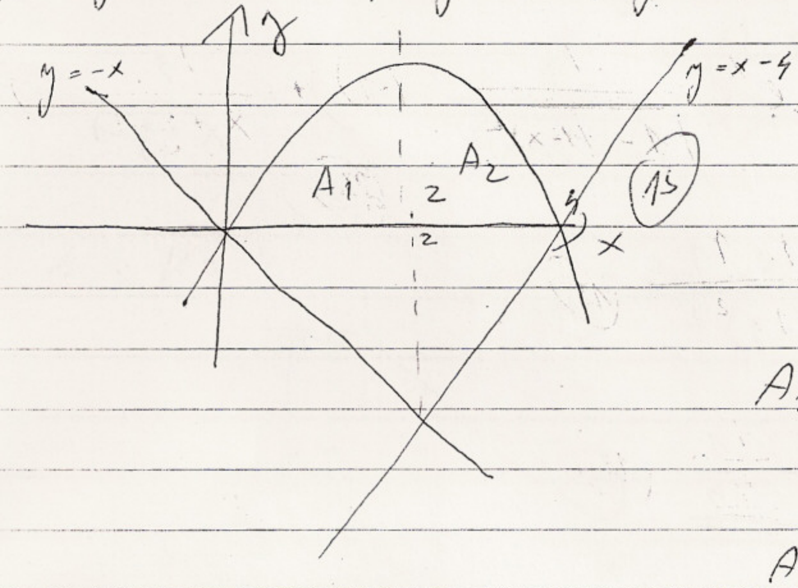
1) $y = 4x - x^2$, $y = -x$, $y = x - 4$



$$y = -x$$
$$y = x - 4$$
$$2y = -4 \Rightarrow y = -2$$
$$x = 2$$

A_1: $0 \le x \le 2$
$-x \le y \le 4x - x^2$ 1,5

A_2: $2 \le x \le 4$
$x - 4 \le y \le 4x - x^2$ 1,0

$P(A) = P(A_1) + P(A_2)$
(alelo $P(A) = 2 \cdot P(A_1)$) 1,5

$$P(A_1) = \int_0^2 [(4x - x^2) - (-x)]\,dx \quad \int_0^2 (4x - x^2 + x)\,dx =$$

$$= \int_0^2 (5x - x^2)\,dx = \left[5\frac{x^2}{2} - \frac{x^3}{3}\right]_0^2 = 5 \cdot \frac{4}{2} - \frac{8}{3} = \frac{30 - 8}{3} =$$

$$= \frac{22}{3} \quad ; \quad \underline{P(A) = \frac{44}{3}}$$

1,5

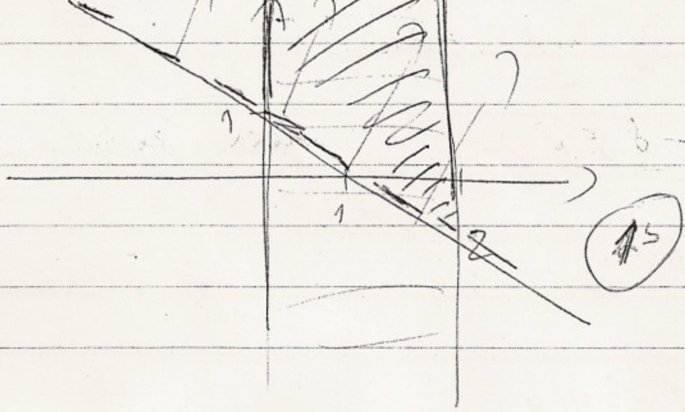
2) $f(x,y) = e^x \cdot \arcsin(1 - x) + \ln(x + y - 1)$

$-1 \le 1 - x \le 1$
$-2 \le -x \le 0$ 1,5
$2 \ge x \ge 0$

$x + y - 1 > 0$ 1,5
$y > 1 - x$

$y = 1 - x$

x	0	1
y	1	0



$D(f) = \{[x,y] \in E_2 : 0 \le x \le 2 \wedge y > 1-x\}$ 1b

$$\frac{\partial f}{\partial x} = e^x \cdot \arcsin(1-x) + e^x \cdot \frac{1}{\sqrt{1-(1-x)^2}} \cdot (-1) + \frac{1}{x+y-1} \cdot 1$$ 2b

$$\frac{\partial^2 f}{\partial x \partial y} = 0 + 0 + \frac{0 - 1 \cdot 1}{(x+y-1)^2}$$ 1b

$$\frac{\partial^2 f}{\partial x \partial y}(A) = \frac{-1}{(1+2-1)^2} = \underline{\underline{-\frac{1}{4}}}$$ 1b

3a) $f(x,y) = x^3 + 2x^2 + y^2 - 2$

$$\frac{\partial f}{\partial x} = 3x^2 + 4x \Rightarrow x \cdot (3x+4) = 0 \Rightarrow x = 0 \vee x = -\frac{4}{3}$$ 1b

$$\frac{\partial f}{\partial y} = 2y = 0 \Rightarrow y = 0$$ 1b

Stac. body: $A[0,0]$, $B[-\frac{4}{3}, 0]$ 1b

$$\frac{\partial^2 f}{\partial x^2} = 6x + 4 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0$$ 1b

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$D = (6x+4) \cdot 2 - 0$$ 4

$D(A) = (0+4) \cdot 2 = 8 > 0$ ... existuje extrém 1b

$\frac{\partial^2 f}{\partial x^2} = 6 \cdot 0 + 4 > 0$ ... lok. min.

$D(B) = (6 \cdot (-\frac{4}{3}) + 4) \cdot 2 = -8 < 0$ ... neex. extrém 1b

b) $\frac{\partial f}{\partial x}(A) = 7$

$\frac{\partial f}{\partial y}(A) = 2$

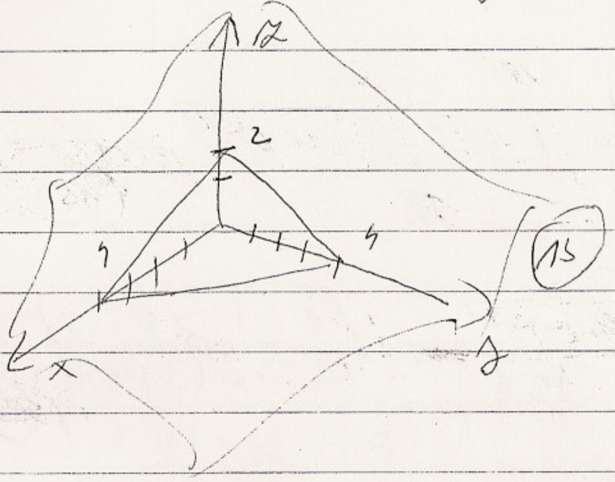
dotyk. ravn.: $t:\ \frac{\partial f}{\partial x}(A)\cdot(x-a_1) + \frac{\partial f}{\partial y}(A)\cdot(y-a_2) = z-a_3$

$$7\cdot(x-1) + 2\cdot(y-1) = z-2$$

$$7x + 2y + z - 7 = 0$$

25

4. $V = ?$ $\quad x+y+2z=4,\ x=0,\ y=0,\ z=0$



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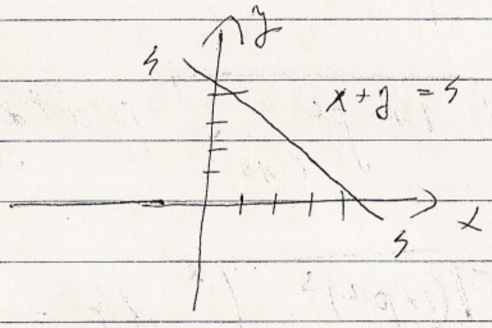

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$$0 \le x \le 4$$

$$0 \le y \le 4-x$$

$$0 \le z \le \frac{4-x-y}{2}$$

15

15

$$V = \iiint_A 1\,dx\,dy\,dz = \int_0^4 dx \int_0^{4-x} dy \int_0^{\frac{4-x-y}{2}} 1\,dz = \int_0^4 dx \int_0^{4-x} \frac{4-x-y}{2}\,dy =$$

15 15

$$= \int_0^4 \left[2y - \frac{x}{2}y - \frac{1}{2}\frac{y^2}{2}\right]_0^{4-x} dx =$$ 15

$$= \int_0^4 \left(2\cdot(4-x) + \frac{x}{2}\cdot(4-x) - \frac{1}{2}\,\frac{16-8x+x^2}{2}\right) dx =$$

$$= \int_0^4 \left(8 - 2x + 2x - \frac{x^2}{2} - 4 + 2x - \frac{x^2}{4}\right) dx =$$

$$= \int_0^4 \left(-\frac{3}{4}x^2 + 2x + 4\right) dx = \left[-\frac{3}{4}\frac{x^3}{3} + 2\frac{x^2}{2} + 4x\right]_0^4 =$$

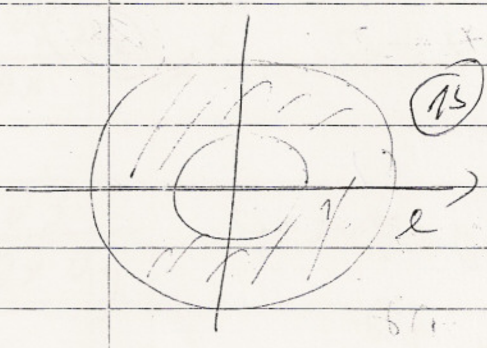
15

$$= -16 + 16 + 16 = \underline{16}$$

5. $\iint_A \sqrt{1+(x^2+y^2)}\,dx\,dy$ $\quad A: 1 \le x^2+y^2 \le e^2$

$0 \le \varphi \le 2\pi$

$1 \le \rho \le e$ (1s)



$= \iint_{A^*} \sqrt{1+\rho^2}$ (1s) $\rho\,d\varphi\,d\rho = \int_0^{2\pi} d\varphi \int_1^e \sqrt{1+\rho^2}\,\rho\,d\rho =$ (1s)

$= \int_0^{2\pi} \left[\frac{\sqrt{(1+\rho^2)^3}}{3}\right]_1^e d\varphi = \int_0^{2\pi}\left(\frac{\sqrt{(1+e^2)^3}}{3} - \frac{\sqrt{8}}{3}\right) d\varphi =$ (2s)

$= \left(\frac{\sqrt{(1+e^2)^3}}{3} - \frac{\sqrt{8}}{3}\right)\cdot 2\pi$

$\int \sqrt{1+\rho^2}\,\rho\,d\rho = \left| \begin{matrix} 1+\rho^2 = t \\ 2\rho\,d\rho = dt \end{matrix} \right| = \frac{1}{2}\int \sqrt{t}\,dt =$

$= \frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{\sqrt{(1+\rho^2)^3}}{3}$ (2s)